Bernoulli Ballot Polling: A Manifest Improvement for Risk-Limiting Audits

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Risk-limiting audits

Statistical check that tabulation errors would not change the electoral outcome.

Risk limit: chance that the audit misses a wrong outcome

https://www.verifiedvoting.org/resources/voting-equipment/

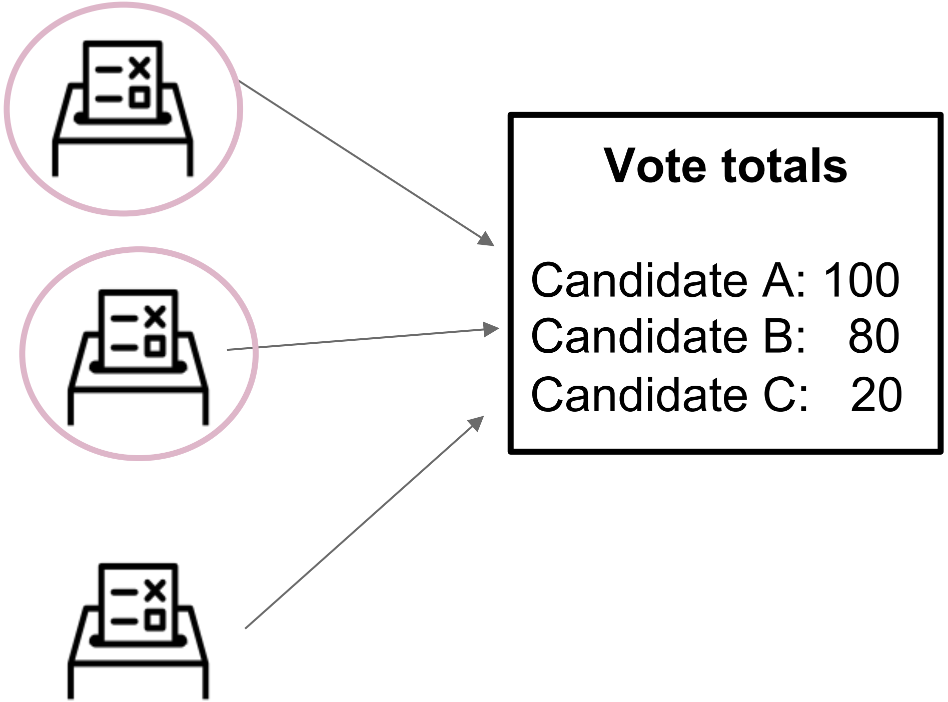
RLAs are hypothesis tests.

H_0: The reported winner is **wrong**.

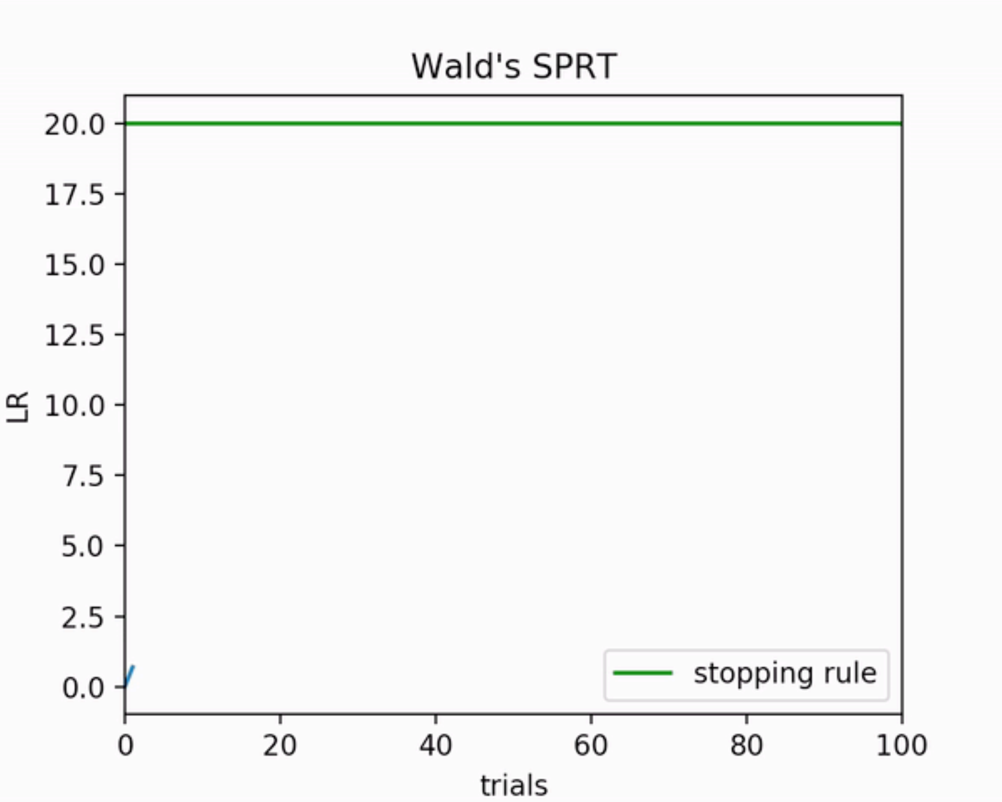
Small p-value = High confidence

Ballot-polling audits

- Sample and tally *n* ballots
- Like drawing M&Ms from a jar to figure out the most common color
- Only need paper record; no extra set-up or inputs from the voting system



Ballot-polling audits



The math is simple, but everything else is hard.

Ballot manifests

Batch ID	Number of Ballots
Election Day Precinct 1	855
Election Day Precinct 2	388
Election Day Precinct 3	702
Election Day Precinct 4	526
Election Day Precinct 5	902
Election Day Precinct 6	941
Election Day Precinct 7	520
Election Day Precinct 8	451

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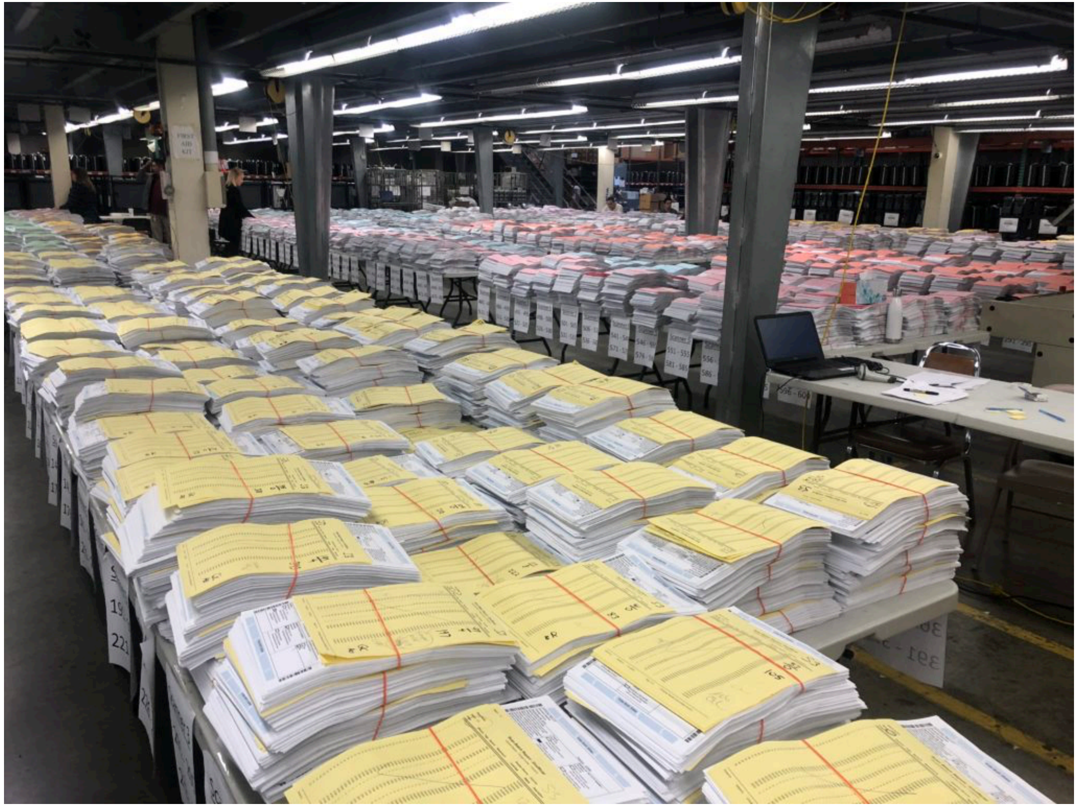
Ballot manifests

Logistics of conducting an audit



https://freedom-to-tinker.com/2018/12/10/pilots-of-risk-limiting-election-audits-in-california-and-virginia/

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Ballot manifests

Logistics of conducting an audit

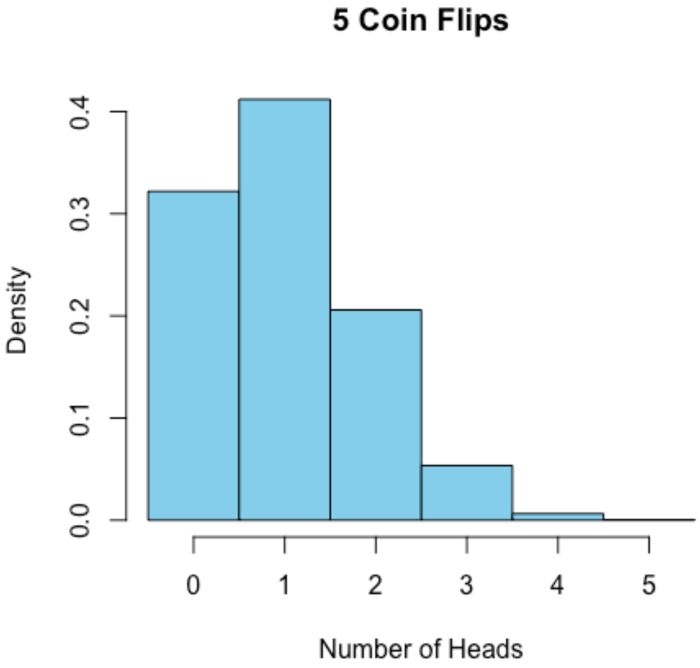
Extra work for local election officials after Election Day

Bernoulli ballot polling (BBP)

Bernoulli sampling

Conceptually, flip a *weighted* coin for each ballot to decide whether it is in the sample.

p=20%



Bernoulli sampling simplifies logistics.

Reduces the need for a ballot manifest

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Work can be conducted “in parallel” **across precincts** on election night

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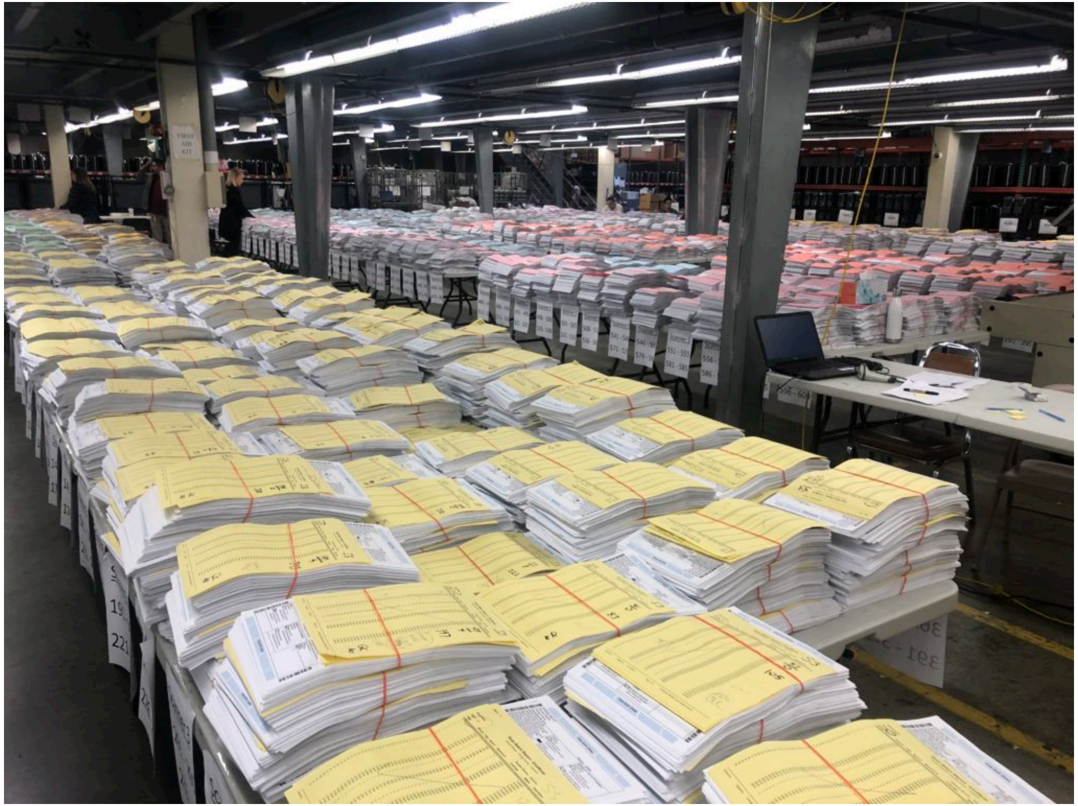


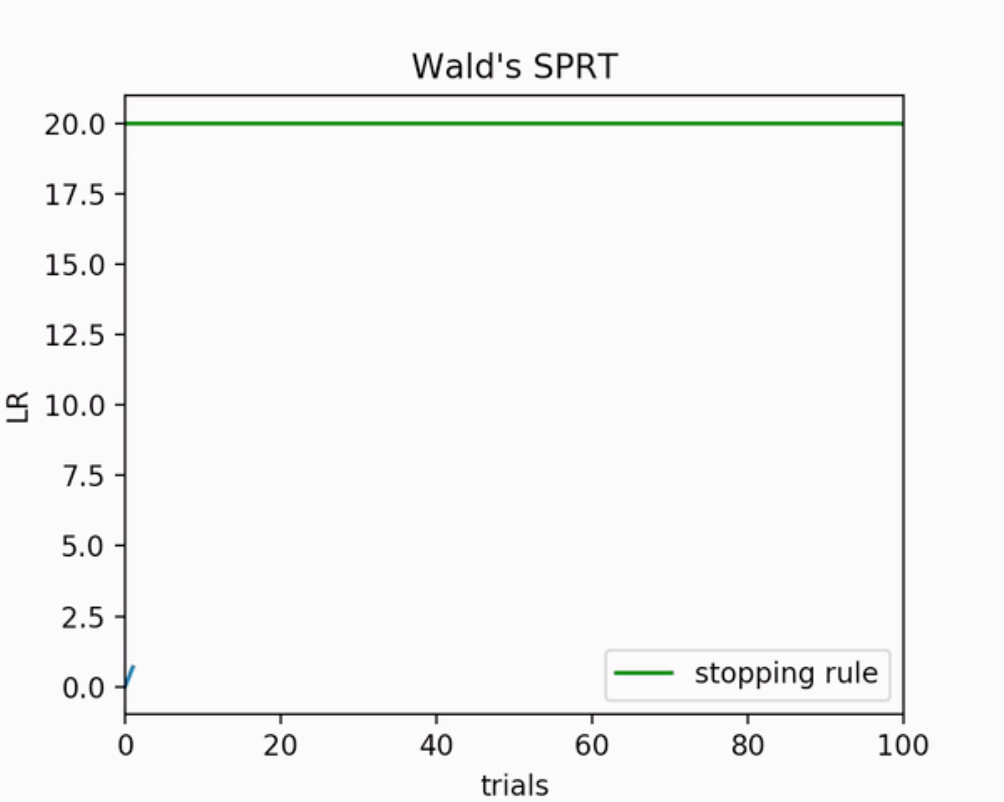
Bernoulli sampling simplifies logistics.

Reduces the need for a ballot manifest

Work can be conducted “in parallel” across precincts on election night

Helps election officials **plan**: set initial sampling rate in advance, estimate labor required

2 key differences that impact the math

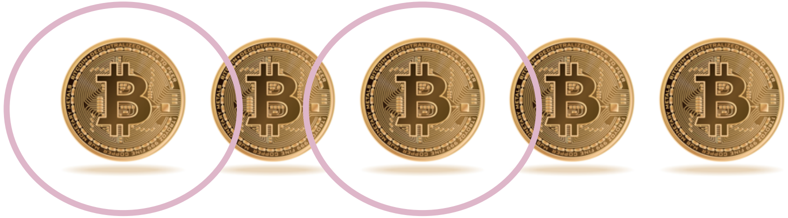


1. Sampling without replacement

2. Sample is decided by flipping a coin for *each* ballot

Bernoulli sampling

Conditional on *n* heads, the sample is a simple random sample:



Any *n* coins are equally likely to land heads.

Bernoulli sampling

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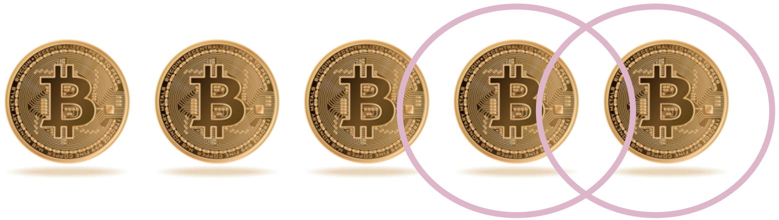
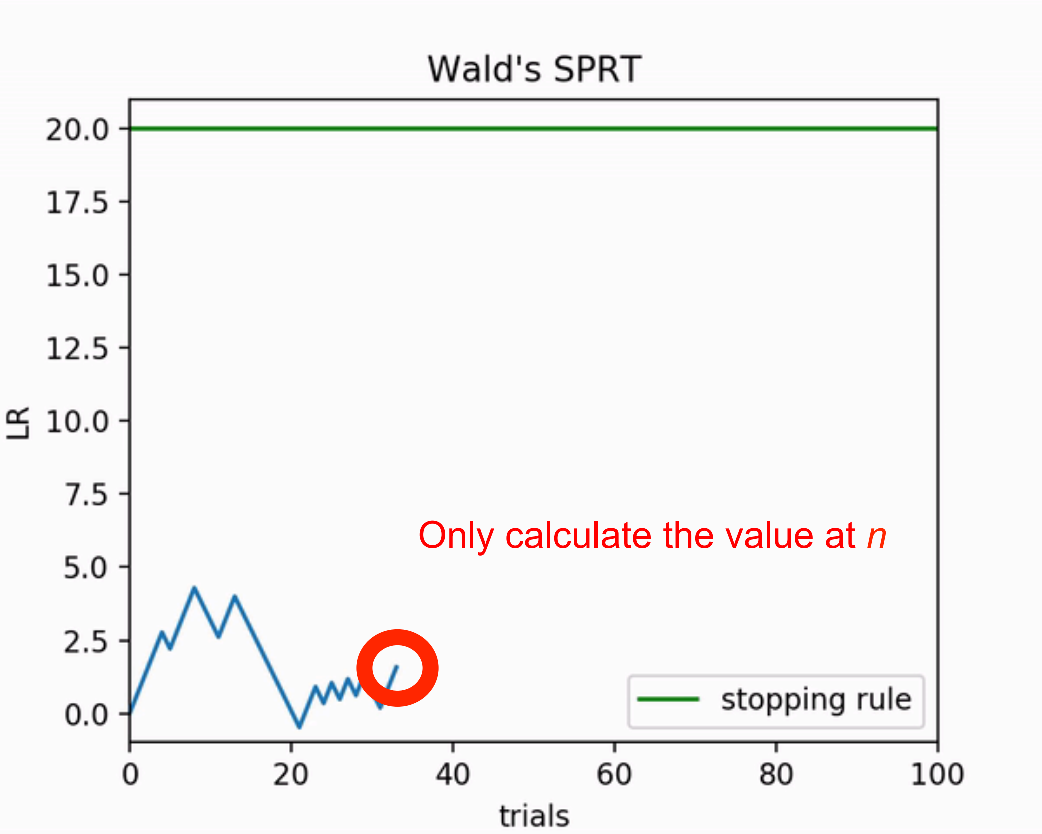


Any *n* coins are equally likely to land heads.

Apply Wald's SPRT to that sample of *n* ballots.



Classical ballot polling ignores ballots *not* for the winner or loser.

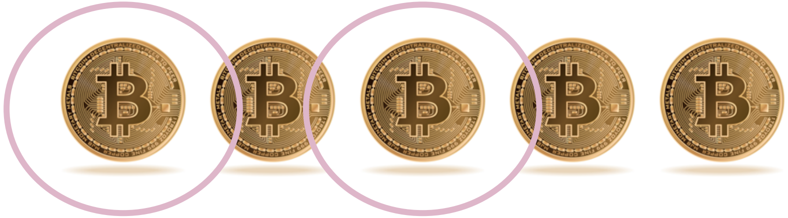
Here, we have to take them into account.

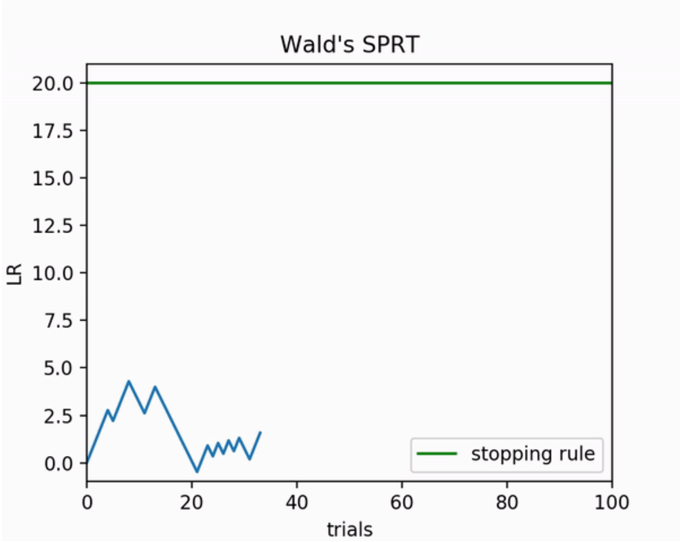
One solution: Maximize the p-value over this *nuisance parameter*.

Implementation

Multiple rounds of sampling

p_0

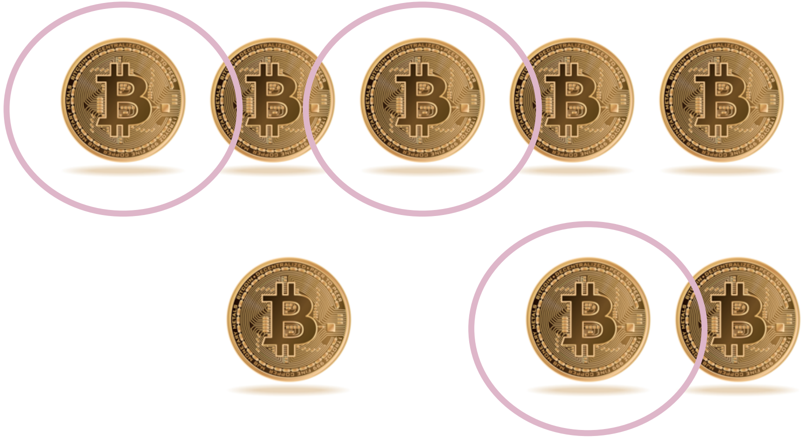


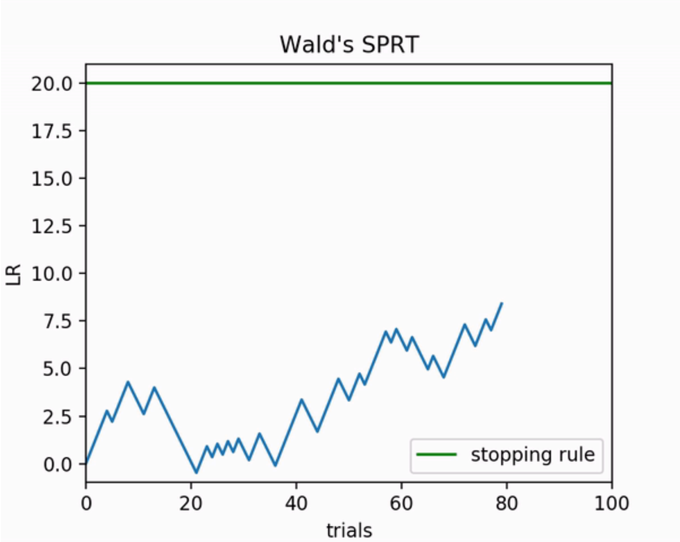


Multiple rounds of sampling

p_0

p_1



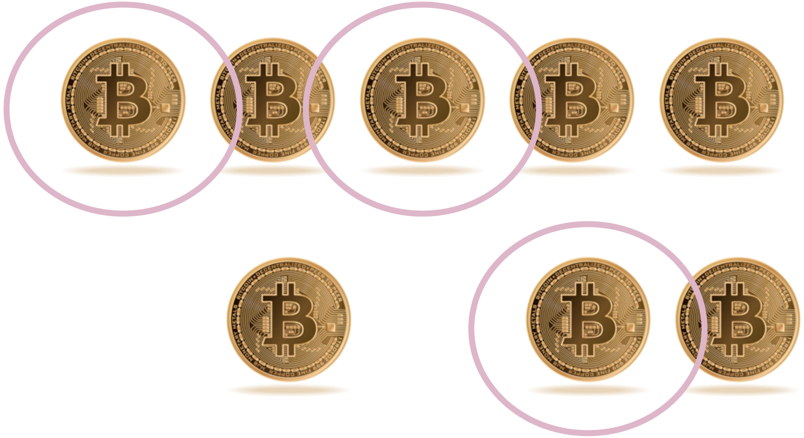


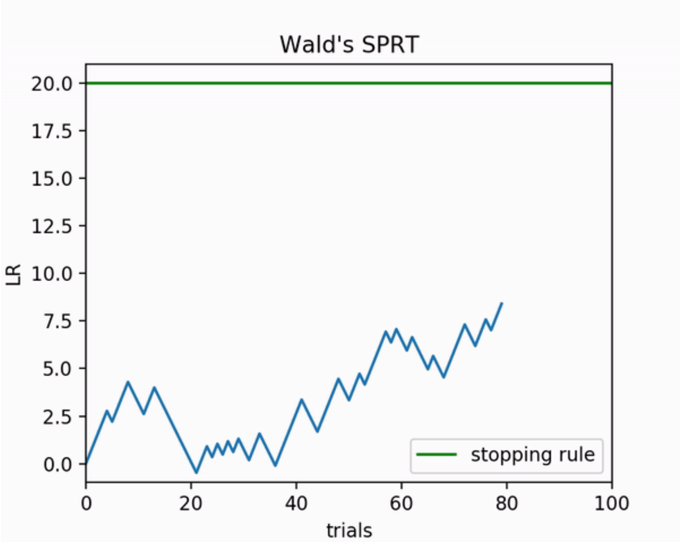
$$1-(1-p_0)(1-p_1)$$

Multiple rounds of sampling

p_0

p_1





$\vdots$

$$p = 1 - \prod_{k=0}^{K-1} (1 - p_k)$$

Initial sampling rate

- Set beforehand at fixed rate or based on expected margins
- Too small could lead to escalation, too large leads to extra work
- Rule of thumb for sample size needed

$$\mathrm{ASN} \approx \frac{2\ln(1/\alpha)}{m^2}$$

Inflate this number if you expect a large fraction of “other” votes.

Efficient coin-flipping: Geometric skipping

The *waiting time* between successive heads of a p coin is a Geometric(p) random variable.

The chance that the next head will be the kth toss after the current head is

$$p(1-p)^{k-1}$$

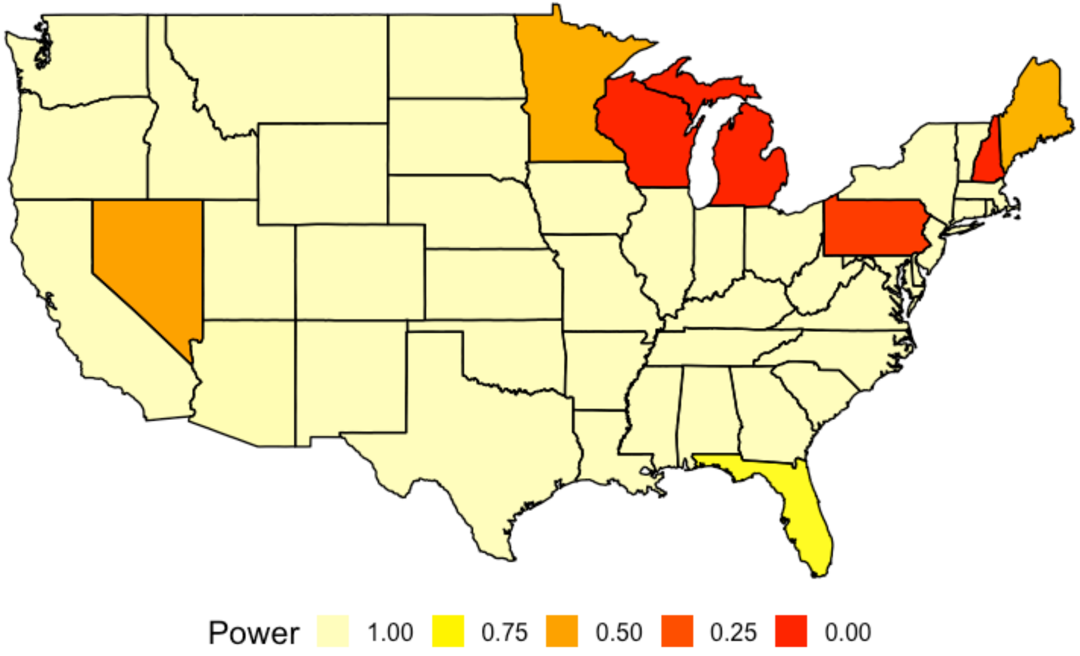
BBP Procedure

1. Set initial sampling rate
2. Sample ballots (using geometric skipping) and record audit data.
3. Check attained risk.
4. Escalate if necessary using more rounds of coin flips.

Bernoulli ballot polling

2016 U.S. Election

1% BBP sample to audit the 2016 presidential election



8 states might need to sample more ballots.

All had margins < 3%.

BBP may have some downsides...

- efficiency depends on choosing the initial p well
- need to train poll workers
- extra work on election day

...but it solves some challenges of extant ballot polling approaches.

- Distributes audit workload
- Similar statistical performance
- Reduces logistical tasks after election day

Thanks!

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